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# The Seiberg-Witten Differential From M-Theory

Ansar Fayyazuddin and Michał Spaliński

*Lyman Laboratory of Physics  
Harvard University  
Cambridge, MA 02138, U.S.A.*

## Abstract

The form of the Seiberg-Witten differential is derived from the M-theory approach to  $N = 2$  supersymmetric Yang-Mills theories by directly imposing the BPS condition for two-branes ending on five-branes. The BPS condition also implies that the pullback of the Kähler form onto the space part of the two-brane world-volume vanishes.

# 1 Introduction

String theory provides a powerful means of analyzing and, at times, solving supersymmetric field theories. It also provides a physical interpretation of the auxiliary geometric structures involved in the solution, which from the field theory point of view appear in a rather mysterious fashion. String theory methods were first used to obtain exact results in  $N = 2$  supersymmetric theories in [1, 2] using the duality of heterotic strings on  $K3 \times T^2$  and type II strings on an appropriate Calabi-Yau threefold. It was soon realized that these results only depend on the local properties of these compactifications, i.e. the ADE singularities of the K3 fiber, which itself could be replaced by an appropriate ALE space. These results were refined further in [3] where it was realized that one could give a very simple picture for the existence of BPS saturated states in the field theory as geodesics around which certain six dimensional self-dual strings could wrap. This was taken a step further in [4] where field theories were constructed using only local properties of type IIA on Calabi-Yau manifolds and performing a local mirror transformation to get the quantum corrected type IIB dual. Some of these developments are reviewed in [5, 6].

Another approach which has proven fruitful is to construct quantum field theories as world-volume theories on branes[7]. Theories with  $N = 2$  supersymmetry in four dimensions have been constructed in terms of type IIA fivebranes in [3]. This description was recently reinterpreted and extended by Witten [8] in M-theory language in a way which brings out the connection with world-volume theories of branes suspended between fivebranes [9, 10]. The solution of the low energy dynamics of these  $N = 2$  supersymmetric quantum field theories [11, 12] can be obtained by appealing to the  $M$ -theory origin[13] of the type IIA string theory. One of the results of this point of view is a geometric interpretation of the field theory beta function. This is the approach which is developed in this note.

The solution of the low-energy dynamics of  $N = 2$  supersymmetric Yang-Mills theory involves the so-called Seiberg-Witten differential. As stressed in [3, 14], in string theory this one-form has a meaning which goes beyond its function in field theory, where it serves merely as the object whose periods give the central charges of the  $N = 2$  supersymmetry algebra (and so in consequence, the relationship between masses and charges of BPS states). While in field theory the Seiberg-Witten differential is defined up to exact

terms, in string theory a particular choice of representative is made. This has led to the identification of BPS states and geodesics [3, 15, 16, 17, 14, 18]. The interpretation of the self-dual string along the geodesic as the boundary of a twobrane ending on a fivebrane is the most natural one from the M-theory point of view explored below.

In this note we show how the Seiberg-Witten differential arises in the M-theory setup proposed in [8]. The picture which emerges is actually very similar to that discussed in [3] in a IIA string theory approach to the  $N=2$  Yang-Mills quantum field theory. In this approach the relationship between type IIB string theory on an ALE space near an  $A_{n-1}$  singularity and type IIA string theory in the presence of  $n$  symmetric fivebranes was used to give a geometric interpretation of the Seiberg-Witten differential in the  $SU(n)$  case. One of the chief virtues of [8] is to give a simple derivation of results in  $N = 2$  supersymmetric quantum field theories which were originally obtained in a more complicated way. In the same spirit, we give a simple derivation of the Seiberg-Witten differential by analyzing the BPS condition directly in M-theory<sup>1</sup>. This argument is valid for all classical groups, as it does not use the explicit form of the Seiberg-Witten curves.

The strategy followed here is to find the BPS states of the  $N = 2$  supersymmetric gauge theory described as M-theory twobranes ending on the fivebrane. The mass of such a state is given by the membrane tension times the area of the membrane. The area form can be found by considering the conditions for preserving supersymmetry, as in [21]. The resulting mass formula can then be compared to the general expression expected on the basis of the Seiberg-Witten analysis [11, 12], which then gives the Seiberg-Witten differential.

## 2 Embedding $N = 2$ supersymmetric Yang-Mills in M-theory

The four dimensional  $N = 2$  supersymmetric  $SU(n)$  Yang-Mills theory can be regarded as the world-volume theory of parallel fourbranes in type IIA theory with finite extent in one direction,  $x^6$ . The ends of the fourbranes, along the  $x^6$  axis, lie on fivebranes. The remaining coordinates of the world-volume

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<sup>1</sup>For a related discussion see [3, 19] and [20].

theory of the fourbranes are  $x^0, x^1, x^2, x^3$ . In the remaining dimensions this “world” is at a definite point.

The fivebranes on which the fourbranes end have world-volumes extending in the  $x^0, x^1, x^2, x^3, x^4, x^5$  coordinates, and each is localized at a point in the remaining coordinates. In the simplest situation there are just two parallel fivebranes found at two finitely separated points in the  $x^6$  direction<sup>2</sup>.

Noting that the fourbrane is just an M-theory fivebrane wrapped around  $x^{10}$ , one can ask what the M-theory configuration of fivebranes would have to be to lead to this type IIA configuration of fourbranes and fivebranes in the weakly coupled limit. The answer[3, 22, 8] is that in M-theory this configuration can be described simply by a *single* M-theory fivebrane with a world-volume  $R^4 \times \Sigma$  where the  $R^4$  here is parameterized by coordinates  $x^0, x^1, x^2, x^3$  and  $\Sigma$  is a Riemann surface of genus  $g = n - 1$  embedded in the Euclidean space  $X$  spanned by  $x^4, x^5, x^6, x^{10}$ . One can endow  $X$  with a complex structure such that  $s = x^6 + ix^{10}$  and  $v = x^4 + ix^5$  are holomorphic and require that  $\Sigma$  is a holomorphic curve in  $X$ . This curve is specified by a holomorphic constraint in  $v$  and  $t \equiv \exp -s/R$  (this is because the  $x^{10}$  coordinate is compactified on a circle of radius  $R$ ). The form of this constraint can be found explicitly by realizing that at any value of  $v$  there should be two fivebranes which remain five branes when the radius  $R$  of  $x^{10}$  is taken to zero. Furthermore, at any value of  $t$  there should be  $n$  fourbranes. These two facts determine  $\Sigma$  to be a holomorphic curve  $F(v, t) = 0$  of degree 2 in  $t$  and degree  $n$  in  $v$ . In the coordinates  $x^7, x^8, x^9$  the fivebrane is localized at some definite point.

Similar considerations apply to the construction of  $\text{SO}(2n)$ ,  $\text{SO}(2n + 1)$ , and  $\text{Sp}(n)$  theories [22, 23, 19, 24]. The main new ingredient is the addition of an orientifold fixed plane parallel to the fourbranes. By working on the covering space this essentially doubles the number of fourbranes by introducing the orientifold images of the fourbranes not localized on the orientifold fixed plane. The net effect from the point of view of M-theory is to restrict the form and degree of the polynomial describing  $\Sigma$  to take into account the positions of the images of the fourbranes.

In the IIA limit one can include matter hypermultiplets [8, 19, 23, 24] by introducing additional fourbranes ending on the “other side” of the IIA

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<sup>2</sup>The picture as described here is accurate only in a certain semiclassical limit, as discussed in [8].

fivebranes. If the fourbranes whose world-volume describes the Yang-Mills theory end on the left of a fivebrane then the matter multiplets end on the right of the fivebrane and vice versa. This configuration can similarly be lifted to an M-theory description by putting appropriate restrictions on the holomorphic curve  $F$  which defines  $\Sigma$  [8, 19, 23, 24].

Thus the picture that emerges is that the brane configurations needed to describe  $N = 2$  supersymmetric gauge theories can be lifted to M-theory as a single fivebrane with world-volume  $x^0, x^1, x^2, x^3, \Sigma$  with  $\Sigma$  a holomorphic curve in  $X$ . These cases will be treated in a unified way below.

### 3 BPS states

On the fivebrane world-volume there is a two-form whose field strength is self-dual. The gauge fields in the four dimensional space  $x^0, \dots, x^3$  are obtained by reducing this self-dual two form on the Riemann surface  $\Sigma$ . This gives rise to  $g = n - 1$  electric  $U(1)$  fields and  $g = n - 1$  magnetic  $U(1)$  duals [25, 8]. BPS states in the fivebrane world-volume theory are twobranes ending on the M-theory fivebrane [26, 27]. The twobrane boundary has to lie on the Riemann surface  $\Sigma$  if they are to represent point particles in the  $x^0, \dots, x^3$  space. The boundary couples to the self-dual two form on the fivebrane world-volume. If the twobrane has a homologically non-trivial boundary then it will couple to the field theory gauge fields obtained by reduction of the self-dual 2-form on that particular homology cycle.

Before analyzing the BPS condition for the twobrane let us discuss the supersymmetry preserving condition for a fivebrane wrapped around a holomorphic curve. The condition for supersymmetric 3-cycles was given in [21] for Calabi-Yau threefold compactifications and has been studied in [28] in the context of two branes wrapping two cycles of a K3.

The number of supersymmetries preserved by a p-brane configuration is given by the number of spinors  $\eta$  which satisfy the equation [21]

$$\eta = \frac{1}{p!} \epsilon^{\alpha_1 \dots \alpha_p} \Gamma_{M_1 \dots M_p} \partial_{\alpha_1} X^{M_1} \dots \partial_{\alpha_p} X^{M_p} \eta, \quad (1)$$

where<sup>3</sup>

$$\Gamma_{M_1 \dots M_p} = \Gamma_{[M_1 \dots M_p]} \quad (2)$$

and  $X^M$  is the embedding of the p-brane in  $R^{9,1} \times S^1$ . Consider first the M-theory fivebrane configuration. For a fivebrane with world-volume filling  $x^0, \dots, x^4, \Sigma$  the supersymmetry condition (in the static gauge) reduces to:

$$\eta = \frac{1}{2} \epsilon^{\alpha\beta} \Gamma_0 \dots \Gamma_3 \Gamma_{ij} \partial_\alpha X^i \partial_\beta X^j \eta, \quad (3)$$

where  $i, j$  label the coordinates  $(X^4, X^5, X^6, X^{10})$ . At this point it is useful to pass to complex coordinates, defined by

$$S = X^4 + iX^5, \quad (4)$$

$$V = X^6 + iX^{10} \quad (5)$$

which we will denote by  $(X^m, X^{\bar{m}})$ .

$$\eta = \frac{1}{2} \epsilon^{\alpha\beta} \Gamma_0 \dots \Gamma_3 (\Gamma_{mn} \partial_\alpha X^m \partial_\beta X^n + \Gamma_{m\bar{n}} \partial_\alpha X^m \partial_\beta X^{\bar{n}} + \Gamma_{\bar{m}\bar{n}} \partial_\alpha X^{\bar{m}} \partial_\beta X^{\bar{n}}) \eta. \quad (6)$$

As the fivebrane is wrapped around the holomorphic curve  $\Sigma$ , only the term with one holomorphic and one anti-holomorphic index is non-vanishing:

$$\eta = \frac{1}{2} \epsilon^{\alpha\beta} \Gamma_0 \dots \Gamma_3 \Gamma_{m\bar{n}} \partial_\alpha X^m \partial_\beta X^{\bar{n}} \eta. \quad (7)$$

From this it follows that

$$\Gamma_0 \dots \Gamma_3 \Gamma_{m\bar{n}} \eta = J_{m\bar{n}} \eta, \quad (8)$$

where  $J$  is the Kähler form on  $X$  ( $J_{m\bar{n}} = ig_{m\bar{n}}$ ). This gives

$$\begin{aligned} i\Gamma_0 \dots \Gamma_3 \Gamma_{V\bar{V}} \eta &= \eta, \\ i\Gamma_0 \dots \Gamma_3 \Gamma_{S\bar{S}} \eta &= \eta, \\ \Gamma_{V\bar{S}} \eta &= 0, \\ \Gamma_{S\bar{V}} \eta &= 0. \end{aligned} \quad (9)$$

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<sup>3</sup>The antisymmetrization in (2) includes a factor of  $1/p!$ , and the epsilon symbol in (1) is a tensor, not a tensor density.

The 11-dimensional spinor  $\eta$  is a 32-component complex spinor, which satisfies the Majorana condition <sup>4</sup>

$$\eta = B\eta^*. \quad (10)$$

It can thus be written as

$$\eta = \chi + B\chi^*. \quad (11)$$

Using

$$B\Gamma_m = -\Gamma_{\bar{m}}B, \quad (12)$$

which follows from the expressions given in the appendix, one finds

$$\begin{aligned} \Gamma_{\bar{V}}\chi &= 0 \\ \Gamma_{\bar{S}}\chi &= 0. \end{aligned} \quad (13)$$

The surviving  $\eta$  has  $32/4 = 8$  complex components. From (9) it then follows that

$$i\Gamma_0 \dots \Gamma_3 \chi = -\chi, \quad (14)$$

which cuts the number of solutions by another factor of 2, so one concludes that there are 4 complex solutions, which confirms that the fivebrane configuration under consideration indeed gives  $N = 2$  supersymmetry in the four dimensional sense.

Introducing the twobrane which ends on the fivebrane requires the additional constraint[26]:

$$\eta = \frac{1}{2}\epsilon^{\alpha\beta}\Gamma_0\Gamma_{ij}\partial_\alpha X^i\partial_\beta X^j\eta, \quad (15)$$

where  $X$  now denotes the embedding of the twobrane. Taking into account equations (9), one finds

$$\begin{aligned} \epsilon_{\alpha\beta}\eta &= \Gamma_0[(\partial_\alpha S\partial_\beta V - \partial_\alpha V\partial_\beta S)\Gamma_{SV} + (\partial_\alpha \bar{S}\partial_\beta \bar{V} - \partial_\alpha \bar{V}\partial_\beta \bar{S})\Gamma_{\bar{S}\bar{V}} \\ &- i(\partial_\alpha S\partial_\beta \bar{S} + \partial_\alpha V\partial_\beta \bar{V} - \partial_\alpha \bar{S}\partial_\beta \bar{S} - \partial_\alpha \bar{V}\partial_\beta V)\Gamma_0 \dots \Gamma_3]\eta. \end{aligned} \quad (16)$$

To analyze this equation it is convenient to define the following projection operators:

$$\begin{aligned} P_+ &= (1 + i\Gamma_0 \dots \Gamma_3)/2, \\ P_- &= (1 - i\Gamma_0 \dots \Gamma_3)/2 \end{aligned} \quad (17)$$

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<sup>4</sup>See the appendix for the definition of  $B$ .

and

$$\begin{aligned} Q_+ &= \Gamma_S \Gamma_{\bar{S}} \Gamma_V \Gamma_{\bar{V}}, \\ Q_- &= (1 - \Gamma_S \Gamma_{\bar{S}} \Gamma_V \Gamma_{\bar{V}}). \end{aligned} \quad (18)$$

These projection operators satisfy a set of simple relations which follow directly from their definition:<sup>5</sup>

$$P_{\pm} \Gamma = \Gamma P_{\mp}, \quad \Gamma \in \{\Gamma_V, \Gamma_S, \Gamma_{\bar{S}}, \Gamma_{\bar{V}}\} \quad (19)$$

and

$$\begin{aligned} P_+ B &= B P_-, & P_+ \Gamma_0 &= \Gamma_0 P_-, \\ Q_+ B &= B Q_-, & Q_{\pm} \Gamma_0 &= \Gamma_0 Q_{\pm}. \end{aligned} \quad (20)$$

From (14) it follows that

$$\begin{aligned} P_+ \chi &= 0, & P_+ B \chi^* &= B \chi^*, \\ P_- \chi &= \chi, & P_- B \chi^* &= 0 \end{aligned} \quad (21)$$

and from (13) one finds:

$$\begin{aligned} Q_+ \chi &= 0, & Q_+ B \chi^* &= B \chi^*, \\ Q_- \chi &= \chi, & Q_- B \chi^* &= 0. \end{aligned} \quad (22)$$

Acting on (16) with  $Q_+$  gives<sup>6</sup>

$$\begin{aligned} \epsilon_{\alpha\beta} B \chi^* &= (\partial_{\alpha} S \partial_{\beta} V - \partial_{\alpha} V \partial_{\beta} S) \Gamma_0 \Gamma_S \Gamma_V \chi \\ &- i(\partial_{\alpha} S \partial_{\beta} \bar{S} + \partial_{\alpha} V \partial_{\beta} \bar{V} - \partial_{\alpha} \bar{S} \partial_{\beta} S - \partial_{\alpha} \bar{V} \partial_{\beta} V) \Gamma_0 B \chi^*. \end{aligned} \quad (23)$$

One can now project the  $P_+$  piece from (23), which implies

$$B \chi^* = \Gamma_0 \Gamma_S \Gamma_V \chi \quad (24)$$

and that the volume form on the spatial part of the two brane world-volume is the pullback of the holomorphic 2-form:

$$\omega = ds \wedge dv. \quad (25)$$

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<sup>5</sup>The relevant properties of the 11 dimensional Dirac algebra are collected in the appendix.

<sup>6</sup>Projection with  $Q_-$  leads to the conjugate equations.



Projecting with  $P_-$  leads to the conclusion that the pullback of the Kähler form to the twobrane must vanish (a similar result was found in a different context in [21]):

$$\partial_\alpha S \partial_\beta \bar{S} + \partial_\alpha V \partial_\beta \bar{V} - \partial_\alpha \bar{S} \partial_\beta S - \partial_\alpha \bar{V} \partial_\beta V = 0. \quad (26)$$

Equation (24) cuts the number of supersymmetries by half, expressing the fact that the membrane state considered here breaks half of the supersymmetry (leaving 4 real supersymmetries). Thus this membrane is a BPS state in the world-volume theory of the fivebrane.

## 4 The Seiberg-Witten Differential

As explained in the introduction, the Seiberg-Witten differential can be inferred from the formula for the mass of a BPS state.

The mass of a BPS saturated twobrane is given by the brane tension times the area:

$$m = T_2 \times (\text{area}) = T_2 \int_{M_2} |\omega|. \quad (27)$$

The last equality follows from the analysis in the previous section where it was found that the area form on the twobrane world-volume is the pullback of the holomorphic two form  $\omega$ . Using the fact that the twobrane has a boundary on  $\Sigma$  one can use Stokes' theorem to write the mass as an integral on the boundary, which in terms of  $t \equiv e^{-s}$  reads

$$m^2 = T_2 \left| \int_{\partial M_2} v(t) \frac{dt}{t} \right|^2. \quad (28)$$

Here  $v(t)$  is  $v$  written as a function of  $t$  on  $\Sigma$ , since the boundary  $\partial M_2$  lies on  $\Sigma$ . Thus the mass of the twobrane reduces to an integral of a meromorphic 1-form on the Riemann surface  $\Sigma$ . From this equation it follows that the Seiberg-Witten differential is:

$$\lambda_{SW} = v(t) \frac{dt}{t}. \quad (29)$$

The form of the Seiberg-Witten differential obtained above is the one which naturally appears in the integrable systems approach  $N = 2$  supersymmetric Yang-Mills [29].

## 5 Conclusions

The analysis of the BPS conditions for a twobrane ending on a fivebrane in the M-theory approach to  $N = 2$  supersymmetric gauge theories leads to two conditions on the twobrane. The first one is that the induced Kähler form on the twobrane vanish, and the second is that the pullback of the holomorphic 2-form  $\omega$  be equal to the area of the twobrane. The latter condition leads directly to the Seiberg-Witten differential. It should also be possible to derive the Seiberg-Witten differential by directly studying the conditions for world-volume supersymmetry in the fivebrane field theory<sup>7</sup>.

## 6 Appendix

This appendix presents some facts concerning the Dirac algebra in 11 dimensions, which are needed to perform the calculations presented in this paper.

The eleven dimensional metric is taken to have signature  $(-, + \dots +)$ .

A representation for the Dirac matrices which is convenient here is given in terms of the following tensor product representation

$$\begin{aligned}\Gamma_\mu &= \gamma_\mu \otimes 1 \otimes 1 & \mu = 0 \dots 3 \\ \Gamma_{i+3} &= \gamma_5 \otimes \tilde{\gamma}_i \otimes 1 & i = 1 \dots 3 \\ \Gamma_{a+6} &= \gamma_5 \otimes \tilde{\gamma}_5 \otimes \sigma_a & a = 1 \dots 3 \\ \Gamma_{10} &= \gamma_5 \otimes \tilde{\gamma}_4 \otimes 1,\end{aligned}\tag{30}$$

where the first and second factors are four-dimensional and the last one is two-dimensional. The individual factors are

$$\gamma_\mu = i \begin{pmatrix} 0 & \sigma_\mu \\ \bar{\sigma}_\mu & 0 \end{pmatrix}\tag{31}$$

and  $\sigma^\mu = (1, \sigma^1, \sigma^2, \sigma^3)$  and  $\bar{\sigma}^\mu = (1, -\sigma^1, -\sigma^2, -\sigma^3)$ , the  $\sigma^i$  being Pauli matrices. Thus  $\gamma^0, \gamma^1, \gamma^3$  are imaginary, while  $\gamma^2$  is real. Furthermore

$$\gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3\tag{32}$$

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<sup>7</sup>This was pointed out to us by C. Vafa.

and

$$\begin{aligned}\tilde{\gamma}_i &= \gamma_i & i = 1 \dots 3; \\ \tilde{\gamma}_4 &= i\gamma_0 \\ \tilde{\gamma}_5 &= \gamma_1\gamma_2\gamma_3\gamma_4.\end{aligned}\tag{33}$$

These choices imply that  $\Gamma_M$  are real for  $M = 2, 5, 7, 8, 10$ , purely imaginary for  $M = 0, 1, 3, 4, 6, 9$ , symmetric for  $M = 0, 2, 5, 7, 8, 10$  and antisymmetric for  $M = 1, 3, 4, 6, 9$ .

The charge conjugation matrix is given by

$$C = B\Gamma_0,\tag{34}$$

where the unitary matrix  $B$  satisfies

$$\Gamma_M^\star = B\Gamma_M B^{-1}\tag{35}$$

and in 11 dimensions one can choose

$$B = B^\star\tag{36}$$

$$B = B^\dagger.\tag{37}$$

In the representation chosen above for the Dirac matrices, we have

$$B = \Gamma_2\Gamma_5\Gamma_7\Gamma_8\Gamma_{10}.\tag{38}$$

The text refers to the matrices

$$\begin{aligned}\Gamma_V &= \frac{1}{2}(\Gamma_4 + i\Gamma_5) & \Gamma_S &= \frac{1}{2}(\Gamma_6 + i\Gamma_{10}) \\ \Gamma_{\bar{V}} &= \frac{1}{2}(\Gamma_4 - i\Gamma_5) & \Gamma_{\bar{S}} &= \frac{1}{2}(\Gamma_6 - i\Gamma_{10})\end{aligned}\tag{39}$$

In the representation chosen here these are purely imaginary.

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